Plan: 1) Mapping class groups
2) Torelli groups
3) Homology
4) Minahan's paper
5) Plan for future talks

1) Mapping class groups

$S = S_g^b$ = compact oriented surface of genus g w/ $b \in \{0,1\}$ boundary components

$\mathrm{Mod}(S) = \pi_0 \mathrm{Diff}^+_\partial(S)$

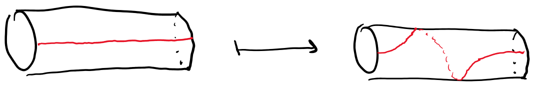
Motivation: $\bullet$ $\mathrm{Diff}^+_\partial(S)$ has contractible path components (Earle - Eells '69)

$\Rightarrow B\mathrm{Diff}^+_\partial(S) \simeq B\mathrm{Mod}(S)$

$\bullet$ $B\mathrm{Mod}(S_g^b) \simeq M_g^b$: moduli space of Riemann surfaces

Generators of Mod(S):

$$T : S^1 \times I \longrightarrow S^1 \times I$$
$$(z, t) \longmapsto (e^{2i\pi t} z, t)$$



$\Rightarrow$ Dehn twist

If γ is a simple closed curve in S and $\varphi : U \longrightarrow S^1 \times I$ is a tubular nbh

let $T_\gamma(x) = \begin{cases} \varphi^{-1} \circ T \circ \varphi(x) & \text{if } x \in U \\ x & \text{o/w} \end{cases}$ Dehn Twist along γ

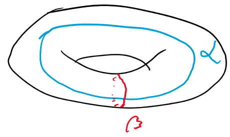
Rk $[T_\gamma] \in \mathrm{Mod}(S)$ only depends on the isotopy class of γ and orientation of φ

Thm [Dehn '38] Mod(S) is generated by finitely many Dehn twists

Thm [Baily '60, Deligne - Mumford '67, McCool '75] Mod(S) is finitely presented

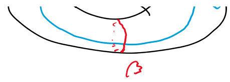
ex $\mathrm{Mod}(S_1) \cong SL(2, \mathbb{Z})$ pf $S_1 \simeq B(\mathbb{Z}^2)$

$T_\alpha \longmapsto \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$



$T_\beta \longmapsto \big(1 \quad 0$

$$T_\alpha \longmapsto \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$



$$T_\beta \longmapsto \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

2) Torelli group

$H_1(S;\mathbb{Z})$ is equipped with an alternating non-degen bilinear form by Poincaré duality (intersection form) symplectic form ω

This symplectic form is invariant under diffeomorphisms

$\Rightarrow \operatorname{Mod}(S) \curvearrowright H_1(S;\mathbb{Z}) \simeq \mathbb{Z}^{2g}$ induces a morphism:

$$\pi : \operatorname{Mod}(S) \longrightarrow Sp(2g,\mathbb{Z}) = \{ g \in GL(2g,\mathbb{Z}) \mid \omega(g\cdot a, g\cdot b) = \omega(a,b) \}$$

Thm π is surjective

Torelli group: $\mathcal{I}(S) = \ker(\pi)$, short exact seq: $1 \to \mathcal{I}(S) \to \operatorname{Mod}(S) \to Sp(2g,\mathbb{Z}) \to 1$

Rk $B\operatorname{Mod}(S) = \mathcal{M}_g^{an}$, $BSp_{2g}(\mathbb{Z}) =$ moduli of ab var

ex $Sp(2,\mathbb{Z}) = SL(2,\mathbb{Z}) \Rightarrow \mathcal{I}(S_1) = 1$

Generators of $\mathcal{I}(S)$

Bounding pair in S = pair (α,β) of simple, closed, non-separating, homologous, non-homotopic curves in S

ex 

Bounding pair map of (α,β) is $T_\alpha^{-1} T_\beta$

Thm [Johnson '83] For $g \geq 3$, $\mathcal{I}(S_g^b)$ is fgen by bounding pair maps

Rk $\mathcal{I}(S_2)$ is not fgen

Open question: Is there a k st for $g \geq k$, $\mathcal{I}(S_k)$ is fin presented?

3) Homology

If G is a group, $H_*(G;\mathbb{Z}) = H_*(BG;\mathbb{Z})$

ex $H_1(G;\mathbb{Z}) \simeq G^{ab}$

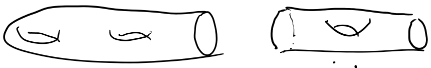
ex $H_1(G;\mathbb{Z}) \cong G^{ab}$

ex If G is f gen, then so is $H_1(G;\mathbb{Z})$ f gen

ex If G is fin pres, then $H_2(G;\mathbb{Z})$ is f gen

$H_*(\mathrm{Mod}(S);\mathbb{Z})$ is very hard!

We have maps $\mathrm{Mod}(S_g^1) \longrightarrow \mathrm{Mod}(S_{g+1}^1)$



Thm [Harer '95] For $g \gg k$, $H_k(\mathrm{Mod}(S_g^1)) \cong H_k(\mathrm{Mod}(S_{g+1}^1))$

Stable homology computed by Madsen-Weiss '07

Q Does $H_k(\mathcal{I}(S_g^1))$ stabilize?

Thm [Johnson '85] For $g \geq 3$, $H_1(\mathcal{I}(S_g^1);\mathbb{Q}) \cong \Lambda^3 H_1(S_g^1)$

$\longrightarrow$ dimension $\binom{2g}{3}$: not eventually constant

Rk $H_k(\mathcal{I}(S_g^1))$ is unknown for $k > 1$

Open Q Is there a k st $\forall g \geq k$, $H_k(\mathcal{I}(S_g))$ is fdim?

Thm [Minahan] For $g \geq 51$, $H_2(\mathcal{I}(S_g);\mathbb{Q})$ is fdim

5) Future talks

1. More background on $\mathrm{Mod}(S)$ and $\mathcal{I}(S)$: Laura + Jules
2. Group homology and cohomology : Clovis + Najib
3. Spectral sequences : Juan + Emmanuel
4. Curve complexes and $\mathcal{C}_{[a]}(S)$: Jules + Muriel
5. Transvective representations : Marie-Camille + Erik
6. The Johnson homomorphism (and abelian cycles) : Adrien

29/11	Laura + Jules	1
6/12	Clovis + Najib	2
13/12	Juan + Emmanuel	3
20/12	Jules + Muriel	4
17/1	Marie-Camille + Erik	5
24/1	Adrien	6
31/1		