R : comm unital noetherian ring (often $R = \mathbb{Q}$)

G : group, $S \subseteq G$ subset, $C_G(S)$ = centralizer of S

Given V an R-module with a G action, and $\mathcal{F} \subseteq G$ a set of pairwise commuting elt

$\forall \mathcal{F}' \subseteq \mathcal{F}$ let $D_G(\mathcal{F}')$ be a subgroup of $C_G(\mathcal{F}')$ such that :

- if $\mathcal{F}'' \subseteq \mathcal{F}'$ then $D_G(\mathcal{F}'') \supseteq D_G(\mathcal{F}')$
- for $\mathcal{F}' = \emptyset$, we take $D_G(\emptyset) = G$

Assume the following are true :

1) for any $\mathcal{F}' \subsetneq \mathcal{F}$ and any $f \in \mathcal{F} \setminus \mathcal{F}'$, then $f \in D_G(\mathcal{F}')$,
   and f normally generates $D_G(\mathcal{F}')$
2) for any $\mathcal{F}' \subsetneq \mathcal{F}$, $H_0(D_G(\mathcal{F}'); V)$ is fgen as R-module
3) $\mathrm{coker}\left(\bigoplus_{f \in \mathcal{F}} V^f \longrightarrow V\right)$ is fgen (V^f : invariants / fixed points)
4) for any $\mathcal{F}' \subsetneq \mathcal{F}$, $D_G(\mathcal{F}')$ has a finite generating set
   contained in $\bigcup_{h \in D_G(\mathcal{F}')} h(\mathcal{F} \setminus \mathcal{F}')h^{-1}$

then $(G, \mathcal{F}, V)$ is called a representation of tranvective type (RTT)

<u>example</u> L = free ab gp equipped with alternating bilinear form $\langle -,- \rangle : L \times L \to \mathbb{Z}$

let $G = Sp(L; \mathbb{Z}) = \{ g \in GL(L) \mid \langle g\cdot v, g \cdot w \rangle = \langle v, w \rangle \ \forall v, w \in L \}$

$\mathcal{F}$ = { primitive transvections along elements of L

where : $v \in L$ is primitive if $\nexists (n \in \mathbb{Z}, w \in L)$ st $n \cdot w = v$

transvection along v = $T_v : L \longrightarrow L$, $w \longmapsto w + \langle v, w \rangle$

<u>ex</u> S : surface of genus g, α : closed simple non-separating curve

$G = Sp(2g; \mathbb{Z})$, $\mathcal{F} = \{ T_{[\alpha]} \}$, $V = H_2(\mathcal{I}_g; \mathbb{Q})$

(Rk : Not very clear in the paper what $D_G(\mathcal{F}')$ is supposed to be ?)
Unclear about condition 1)

<u>prop</u> If $(G, \mathcal{F}, V)$ is an RTT then V is finitely generated

<u>proof</u> By induction on $|\mathcal{F}|$

- If $|\mathcal{F}| = 1$: by hyp 4, $D_G(\emptyset)$ is fin gen by a set $\mathcal{X}$ of conjugates of f

* If $\underset{F=\{f\}}{|F|=1}$: by hyp 4, $D_G(\emptyset)$ is fin gen by a set $\mathcal{H}$ of conjugates of f

Let $\varphi : V \longrightarrow \bigoplus_{g\in\mathcal{H}} V/V^g$: let us show that $\ker\varphi$ & $\operatorname{im}\varphi$ are fin gen

<u>$\operatorname{im}\varphi$</u> ; every $g\in\mathcal{H}$ is of the form $g = h_g f h_g^{-1}$

$\Rightarrow h_g(-)h_g^{-1} : V/V^f \longrightarrow V/V^g$ iso

by hyp 2, V/V^f is fin gen $\Rightarrow V/V^g$ as well

$\Rightarrow \bigoplus_{g\in\mathcal{H}} V/V^g$ is fin gen $\Rightarrow \operatorname{im}\varphi$ is fin gen

<u>$\ker\varphi$</u> : $\ker\varphi = \bigcap_{g\in\mathcal{H}} V^g$ is a trivial G-module (b/c $\mathcal{H}$ generates G)

LES: $\cdots \longrightarrow H_1(G;\operatorname{im}\varphi) \longrightarrow H_0(G;\ker\varphi) \longrightarrow H_0(G;V) \longrightarrow H_0(G,\operatorname{im}\varphi) \to 0$

$\left.\begin{array}{ll}\text{by hyp 4 :} & H_1(G;\operatorname{im}\varphi) \text{ fin gen} \\ \text{hyp 2 :} & H_0(G;V) \text{ fin gen}\end{array}\right\} \Rightarrow H_0(G;\ker\varphi)$ fin gen

but the action is trivial $\Rightarrow H_0(G;\ker\varphi) = \ker\varphi$ is fin gen

* Induction : $|F|>1$, let $f\in F \Rightarrow$ right exact sequence:

$$\bigoplus_{g\in F\setminus\{f\}} V^g \longrightarrow V \longrightarrow V' \longrightarrow 0$$

let $G_f = D_G(F\setminus\{f\})$

<u>lemma</u> $(G_f, \{f\}, V')$ is an RTT

1) we need to show that f normally generates G_f
   $\hookrightarrow$ true by hyp 1 applied to (G,F,V) and $F' = F\setminus\{f\}$
2) there is a surjection : $H_0(\underbrace{G_f}_{=D_G(F\setminus\{f\})};V) \twoheadrightarrow H_0(G_f;V')$ $\Rightarrow$ fin gen by hyp 2
3) $\operatorname{coker}\left(\bigoplus_{g\in G} V^g \to V\right) \twoheadrightarrow \operatorname{coker}\left((V')^f \to V'\right)$
   $\Rightarrow$ fin gen by hyp 3
4) true by hyp 4 applied to $G_f = D_G(F'\setminus\{f\})$

It follows by the inductive hyp that V' is f.gen

<u>lemma</u> $(G, F\setminus\{f\}, V)$ is an RTT

| cond 1, 2, 4 are obvious

Lemma $(G, F \setminus \{f\}, V)$ is an RTT

- cond 1, 2, 4 are obvious
- cond 3: coker $\left(\bigoplus_{g \in F \setminus \{1\}} V^g \longrightarrow V\right)$ is V' from before
  $\Rightarrow$ fin gen

$\Rightarrow$ by induction, V is fgen QED

Recall from Laura's talk.

Lemma $g \geq 2$, $b \geq 0$, $S = S_g^b$, $M \subseteq S$ nonseparating multicurve st $g(S \setminus M) \geq 1$
let G be the image of $\mathrm{Mod}(S \setminus M) \longrightarrow \mathrm{Mod}(S) \longrightarrow \mathrm{Aut}(H_1(S_g^b; \mathbb{Z}))$
then: 1) G has a fin gen set consisting of transvections
2) the G-normal closure of any primitive transvection is G