R : comm unital noetherian ring (often $R = \mathbb{Q}$)

G : group, $S \subseteq G$ subset, $C_G(S)$ = centralizer of S

Given V an R-module with a G action, and $F \subseteq G$ a set of pairwise commuting elt

$\forall F' \subseteq F$ let $D_G(F')$ be a subgroup of $C_G(F')$ such that :

- if $F'' \subseteq F'$ then $D_G(F'') \supseteq D_G(F')$
- for $F' = \emptyset$, we take $D_G(\emptyset) = G$

Assume the following are true :

1) for any $F' \subsetneq F$ and any $f \in F \setminus F'$, then $f \in D_G(F')$, and f normally generates $D_G(F')$
2) for any $F' \subsetneq F$, $H_0(D_G(F'); V)$ is fgen as R-module
3) $\operatorname{coker}\left(\bigoplus_{f \in F} V^f \longrightarrow V\right)$ is fgen (V^f: invariants / fixed points)
4) for any $F' \subsetneq F$, $D_G(F')$ has a finite generating set contained in $\bigcup_{h \in D_G(F')} h(F \setminus F')h^{-1}$

then (G, F, V) is called a representation of tranvective type (RTT)

<u>example</u> L = free ab gp equipped with alternating bilinear form $\langle -,- \rangle : L \times L \to \mathbb{Z}$

let $G = Sp(L; \mathbb{Z}) = \{ g \in GL(L) \mid \langle g\cdot v, g\cdot w \rangle = \langle v, w \rangle \ \forall v, w \in L \}$

F = {primitive transvections along elements of L

where : $v \in L$ is primitive if $\nexists (n \in \mathbb{Z}, w \in L)$ st $n \cdot w = v$

transvection along v = $T_v : L \longrightarrow L$, $w \longmapsto w + \langle v, w \rangle$

<u>ex</u> S: surface of genus g, α : closed simple non-separating curve

$G = Sp(2g; \mathbb{Z})$, $F = \{T_{[\alpha]}\}$, $V = H_2(\mathcal{I}_g; \mathbb{Q})$

(<u>Rk</u> : Not very clear in the paper what $D_G(F')$ is supposed to be ?)
Unclear about condition 1)

<u>prop</u> If (G, F, V) is an RTT then V is finitely generated

<u>proof</u> By induction on $|F|$

* If $|F| = 1$: by hyp 4, $D_G(\emptyset)$ is fin gen by a set X of conjugates of f

* If $\underset{F=\{f\}}{|F|=1}$: by hyp 4, $D_G(\emptyset)$ is fin gen by a set $\mathcal{H}$ of conjugates of f

Let $\varphi : V \longrightarrow \bigoplus_{g\in\mathcal{H}} V/V^g$: let us show that $\ker\varphi$ & $\operatorname{im}\varphi$ are fin gen

<u>$\operatorname{im}\varphi$</u> ; every $g\in\mathcal{H}$ is of the form $g = h_g f h_g^{-1}$

$\Rightarrow h_g(-)h_g^{-1} : V/V^f \longrightarrow V/V^g$ iso

by hyp 2, V/V^f is fin gen $\Rightarrow V/V^g$ as well

$\Rightarrow \bigoplus_{g\in\mathcal{H}} V/V^g$ is fin gen $\Rightarrow \operatorname{im}\varphi$ is fin gen

<u>$\ker\varphi$</u> : $\ker\varphi = \bigcap_{g\in\mathcal{H}} V^g$ is a trivial G-module (b/c $\mathcal{H}$ generates G)

LES: $\cdots \longrightarrow H_1(G;\operatorname{im}\varphi) \longrightarrow H_0(G;\ker\varphi) \longrightarrow H_0(G;V) \longrightarrow H_0(G,\operatorname{im}\varphi) \to 0$

$$\left.\begin{array}{ll}\text{by hyp 4 :} & H_1(G;\operatorname{im}\varphi) \text{ fin gen}\\ \text{hyp 2 :} & H_0(G;V) \text{ fin gen}\end{array}\right\} \Rightarrow H_0(G;\ker\varphi) \text{ fin gen}$$

but the action is trivial $\Rightarrow H_0(G;\ker\varphi) = \ker\varphi$ is fin gen

* Induction : $|F|>1$, let $f\in F \Rightarrow$ right exact sequence:

$$\bigoplus_{g\in F\setminus\{f\}} V^g \longrightarrow V \longrightarrow V' \longrightarrow 0$$

let $G_f = D_G(F\setminus\{f\})$

<u>lemma</u> $(G_f, \{f\}, V')$ is an RTT

1) we need to show that f normally generates G_f

$\hookrightarrow$ true by hyp 1 applied to (G,F,V) and $F' = F\setminus\{f\}$

2) there is a surjection : $H_0(\underbrace{G_f}_{=D_G(F\setminus\{f\})};V) \twoheadrightarrow H_0(G_f;V')$ $\Rightarrow$ fin gen by hyp 2

3) $\operatorname{coker}\left(\bigoplus_{g\in G} V^g \to V\right) \twoheadrightarrow \operatorname{coker}\left((V')^f \longrightarrow V'\right)$

$\Rightarrow$ fin gen by hyp 3

4) true by hyp 4 applied to $G_f = D_G(F'\setminus\{f\})$

It follows by the inductive hyp that V' is f.gen

<u>lemma</u> $(G, F\setminus\{f\}, V)$ is an RTT

cond 1, 2, 4 are obvious

<u>lemma</u> $(G, F\setminus\{1\}, V)$ is an RTT

- cond 1, 2, 4 are obvious
- cond 3: $\operatorname{coker}\left(\bigoplus_{g\in F\setminus\{1\}} V^g \longrightarrow V\right)$ is V' from before
  $\Rightarrow$ fin gen

$\Rightarrow$ by induction, V is fgen QED

Recall from Laura's talk.

<u>lemma</u> $g \geq 2$, $b \geq 0$, $S = S_g^b$, $M \subseteq S$ nonseparating multicurve st $g(S \setminus M) \geq 1$

let G be the image of $\operatorname{Mod}(S \setminus M) \longrightarrow \operatorname{Mod}(S) \longrightarrow \operatorname{Aut}(H_1(S_g^b; \mathbb{Z}))$

then: 1) G has a fin gen set consisting of transvections

2) the G-normal closure of any primitive transvection is G